Class 7 - Lines

Review:

→ **Operations with fractions**

$$\frac{1}{6} - \frac{2}{3} \cdot \frac{2}{2} = \frac{1}{6} - \frac{4}{6} = \frac{1-4}{6} = -\frac{3}{6} \begin{matrix} \div 3 \\ \div 3 \end{matrix} = -\frac{1}{2}.$$

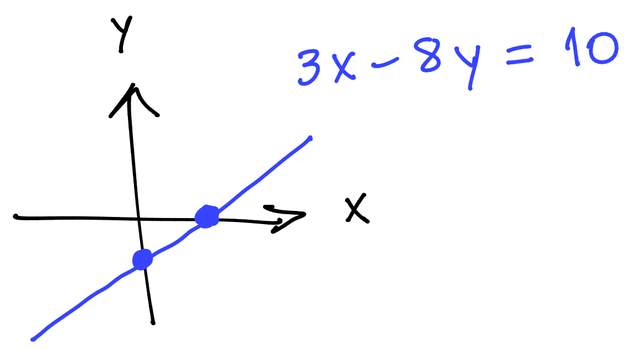
On calculator: $1 \div 6 - 2 \div 3 \longrightarrow -0.5$ 2nd, PRB $\longrightarrow -\frac{1}{2}$ (2nd, A b/c)

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8} \longrightarrow (1 \div 2) \times (3 \div 4)$$

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{1 \cdot 4}{2 \cdot 3} = \frac{4}{6} \begin{matrix} \div 2 \\ \div 2 \end{matrix} = \frac{2}{3} \longrightarrow (1 \div 2) \div (3 \div 4)$$

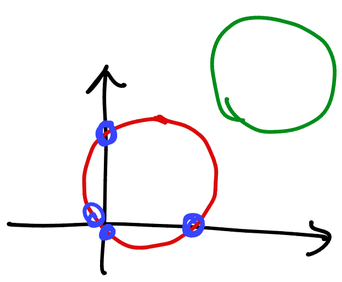
→ **Finding intercepts**

$3x - 8y = 10$



x-intercept: set $y = 0$, $3x - 8 \cdot 0 = 10 \Rightarrow \frac{3x}{3} = \frac{10}{3} \Rightarrow \boxed{x = \frac{10}{3}}$

y-intercept: set $x = 0$. $3 \cdot 0 - 8y = 10 \Rightarrow \frac{-8y}{-8} = \frac{10}{-8} \Rightarrow \boxed{y = -\frac{5}{4}}$



→ Rearrange a Linear equation (in terms of "x")
("solving" for y)

$3x - 8y = 7 \rightarrow -8y = -3x + 7 \rightarrow y = \frac{3}{8}x - \frac{7}{8}$

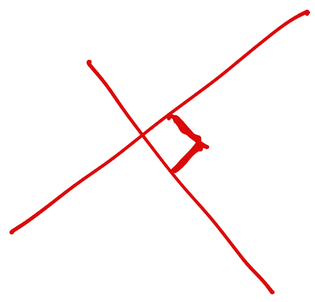
(−3x, −3x; −8, −8)

slope

→ Reciprocals & Negative Reciprocal

- $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals $\Leftrightarrow \frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$.

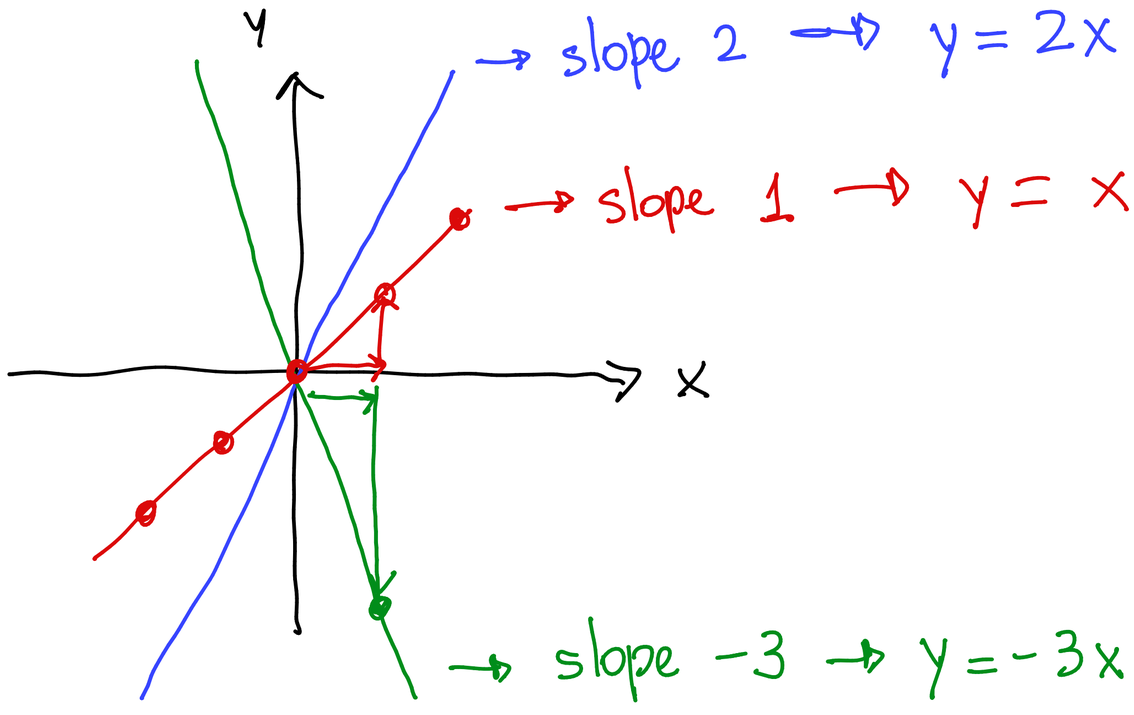
- $-\frac{2}{5}$ and $\frac{5}{2}$ are negative reciprocals $\Leftrightarrow -\frac{2}{5} \cdot \frac{5}{2} = -\frac{2 \cdot 5}{5 \cdot 2} = -\frac{10}{10} = -1$.



the slopes are

N.R.

Lines



A point (x, y) belongs to a line if, and only if its equation is true for (x, y).

Consider $2x - y = 5$. Is the point $(\overset{x}{2}, \overset{y}{3})$ on this line?

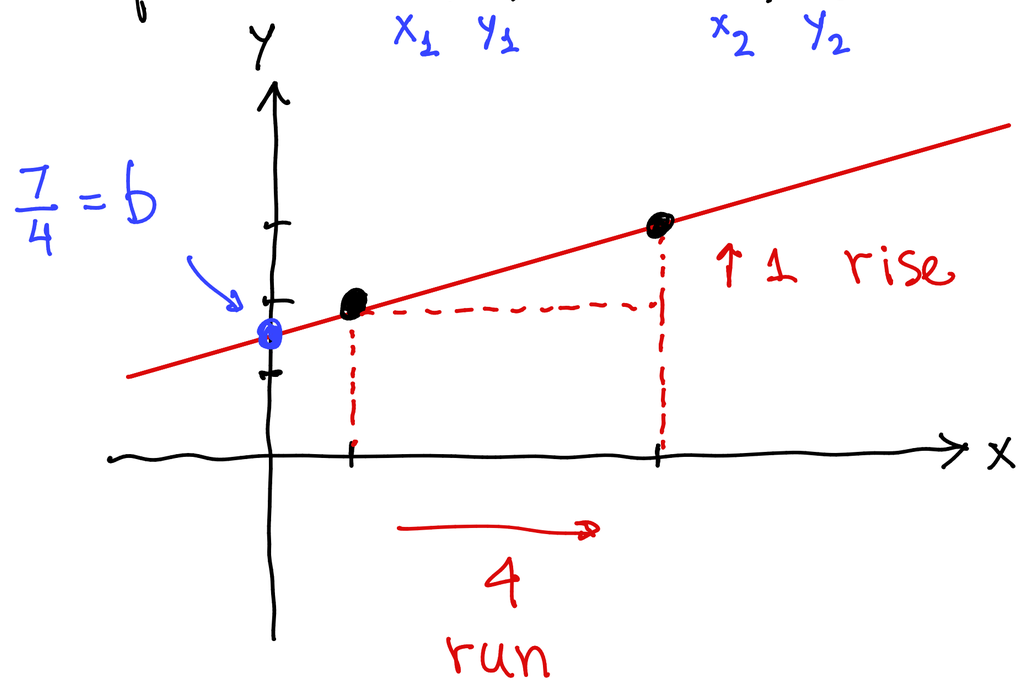
$2 \cdot 2 - 3 = 5 \Leftrightarrow 4 - 3 = 5$ FALSE $\Leftrightarrow$ (2, 3) is not a point on the line $2x - y = 5$.

→ check that (2, −1) is on the line.

→ How to compute the slope

Ex1: Determine the slope of the line passing at the points $(1,2)$ & $(5,3)$.

$x_1\ y_1$ $\qquad$ $x_2\ y_2$



$$y = mx + b$$

slope $\quad$ y-int

slope $= \frac{1}{4}$

1st: Compute slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2nd: plug a point to find b

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{3-2}{5-1} = \frac{1}{4}.$$

$$y = \frac{1}{4}x + b$$

Check for $(1,2)$: $\quad 2 = \frac{1}{4} \cdot 1 + b \iff 2 = \frac{1}{4} + b$

$-\frac{1}{4} \qquad -\frac{1}{4}$

$$4 \cdot \frac{2}{1} - \frac{1}{4} = \frac{8}{4} - \frac{1}{4} = \frac{7}{4}$$

$$\iff \boxed{\frac{7}{4} = b}$$

Check for $(5,3)$: $\quad 3 = \frac{1}{4} \cdot 5 + b \iff 3 = \frac{5}{4} + b$

$-\frac{5}{4} \qquad -\frac{5}{4}$

$$\iff 4 \cdot \frac{3}{1} - \frac{5}{4} = b$$

$$\iff \frac{12}{4} - \frac{5}{4} = b$$

$$\iff \boxed{\frac{7}{4} = b}$$

the equation $y = mx + b$ is called slope-intercept form

Ex 2: Find the equation of the line passing through the points $(4,-2)$ and $(1,3)$.

$(4,-2)$: x_1, y_1 — $(1,3)$: x_2, y_2

1st : slope = m

2nd : y-int. = b

3rd : write equation

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{1-4} = \frac{3+2}{-3} = -\frac{5}{3}.$$

$y = -\frac{5}{3}x + b$. Plug-in $(\underset{x}{1}, \underset{y}{3})$:

$$3 = -\frac{5}{3}\cdot 1 + b \iff 3 = -\frac{5}{3} + b \iff \frac{5}{3} + 3 = b$$

$+\frac{5}{3}$ $+\frac{5}{3}$

$$b = \frac{5}{3} + \frac{9}{3} \iff b = \frac{14}{3}.$$ Equation: $\boxed{y = -\frac{5}{3}x + \frac{14}{3}}$

Ex 3 : find the equation of the line passing at $(-1,2)$ and $(2,5)$.

$m = 1$ & $b = 3$.

Finding the equation of a line given the slope & a point:

Ex3: Find the equation, given $m = 1$ & $\underset{x_0}{(-1}, \underset{y_0}{2})$. $\boxed{y = x + 3}$

$$m = \frac{y - y_0}{x - x_0} \iff \boxed{y - y_0 = m(x - x_0)} \implies y - 2 = 1(x+1)$$

or $y - 2 = x + 1$

point-slope form

Ex 4: Given $m = \frac{2}{3}$ and the point $\underset{x_0}{(-1}, \underset{y_0}{2})$. Find the equation of this line using the point-slope form.

$$y - y_0 = m(x - x_0) \longrightarrow y - 2 = \frac{2}{3}(x - (-1)) \iff \boxed{y - 2 = \frac{2}{3}(x+1)}$$

$$y - \underset{+2}{2} = \frac{2}{3}x + \frac{2}{3} + 2 \iff \boxed{y = \frac{2}{3}x + \frac{8}{3}} \longrightarrow \text{slope-intercept form}$$

Standard Form

$Ax + By = C$, $A > 0$ (only use integers)

Ex 1: Write the equation $y = \frac{2}{3}x + \frac{8}{3}$ in standard form.

$3(y) = \left(\frac{2}{3}x + \frac{8}{3}\right)3 \iff 3y = \frac{2}{3}x \cdot 3 + \frac{8}{3} \cdot 3 \iff 3y = 2x + 8$

$-2x \quad -2x$

$-2x + 3y = 8 \longrightarrow \boxed{2x - 3y = -8}$

Exercise: write in all different forms the equation of the line with slope $m = 4$ and y-intercept $b = -3$.

$x = 1 \Rightarrow y = 4 \cdot 1 - 3 = 1$ $\quad (1, 1)$

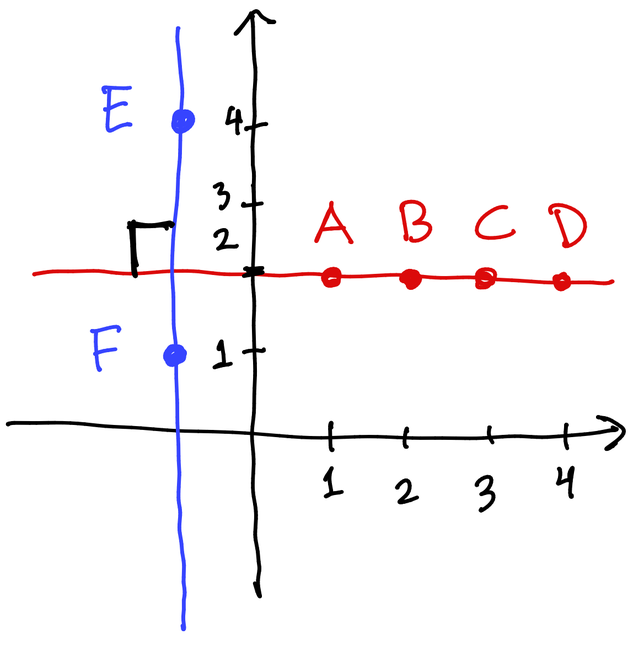
→ slope-int.: $y = mx + b \rightarrow y = 4x - 3$

→ point-slope: $y - y_0 = m(x - x_0) \rightarrow y - 1 = 4(x - 1)$

→ std form: $Ax + By = C \rightarrow y = 4x - 3 \rightarrow y - 3 = 4x \rightarrow 4x - y = -3$

$+3 \quad +3 \qquad -y \quad -y$

Vertical & Horizontal Lines



$A = (1, 2)$ $\quad B = (2, 2)$ $\quad C = (3, 2)$ $\quad D = (4, 2)$

$\boxed{y = 2}$

$E = (-1, 4)$ $\quad F = (-1, 1)$ $\quad \boxed{x = -1}$

Parallel & Perpendicular Lines

Theorem 1: two lines are parallel iff their slopes are equal.

Theorem 2: two lines are perpendicular iff their slopes are negative reciprocals.

Ex 1: Verify if the lines $8x+2y=6$ and $3x-12y=9$ are parallel, perpendicular or neither.

$$8x+2y=6 \Rightarrow \frac{2y}{2}=\frac{-8x+6}{2} \Rightarrow y=-4x+3 \rightarrow m=-\frac{4}{1}$$
$$-8x \qquad -8x$$

$$3x-12y=9 \Rightarrow \frac{-12y}{-12}=\frac{-3x+9}{12} \Rightarrow y=-\frac{1}{4}x+\frac{3}{4} \rightarrow m=-\frac{1}{4}$$
$$-3x \qquad -3x$$

The lines are neither.

Ex 2: Verify if the lines $2x+3y=5$ and $6x+9y=10$ are parallel, perpendicular or neither.

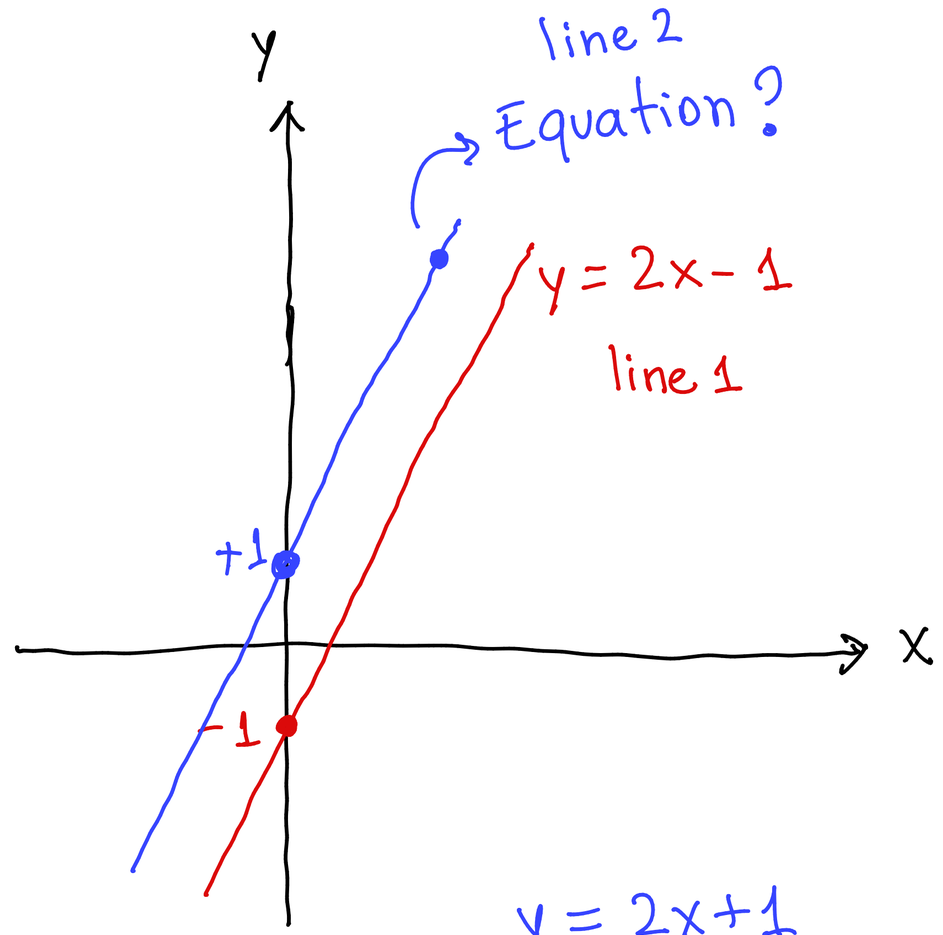
$$2x+3y=5 \rightarrow \frac{3y}{3}=\frac{-2x+5}{3} \rightarrow y=-\frac{2}{3}x+\frac{5}{3} \rightarrow m=-\frac{2}{3}$$
$$-2x \qquad -2x$$

$$6x+9y=10 \rightarrow \frac{9y}{9}=\frac{-6x+10}{9} \rightarrow y=-\frac{2}{3}x+\frac{10}{9} \rightarrow m=-\frac{2}{3}$$
$$-6x \qquad -6x$$

These lines are parallel.

Find equation of Perp. or Par. Lines

Exercise: consider the equation $y = 2x - 1$. Find the equation of the parallel to this line, but passing through the point $(2,5)$.



1st: Use theorems 1 & 2 to get the slope of line 2

→ slope of line 1 = slope of line 2

eq. line 2 → $y = 2x + b$

2nd: use the given point to determine the equation.

$y - y_0 = m(x - x_0) \rightarrow y - 5 = 2(x-2) \rightarrow y - 5 = 2x - 4$

$y = 2x + 1$

Exercise: consider the equation $6x - 5y = 4$. Find the equation of the perpendicular to this line, but passing through the point $(3,-5)$. Write it in point-slope form.

1st: perpendicular $\Rightarrow$ neg. reciprocal.

$$\underset{-6x}{6x - 5y} = \underset{-6x}{4} \Rightarrow \frac{-5y}{-5} = \frac{-6x+4}{-5} \Rightarrow \boxed{y = \frac{6}{5}x - \frac{4}{5}}$$

line 1 has slope $m = \frac{6}{5}$

line 2 has slope $\boxed{m = -\frac{5}{6}}$

x_0 y_0
$(3,-5)$

2nd: use the pt to get equation:

$y - y_0 = m(x - x_0) \rightarrow y - (-5) = -\frac{5}{6}(x-3) \rightarrow \boxed{y + 5 = -\frac{5}{6}(x-3)}$